Christ Church Grammar School

2021
TEST 2

MATHEMATICS SPECIALIST Year 12
Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	25 minutes
Marks available:	25 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet using a blue/black pen. Do not use erasable or gel pens.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(7 marks)**

Functions f and g are defined such that: $f(x) = \sqrt{4-x}$ $g(x) = \frac{2}{x-2}$

(a) Determine $gf(x)$. (1 mark)

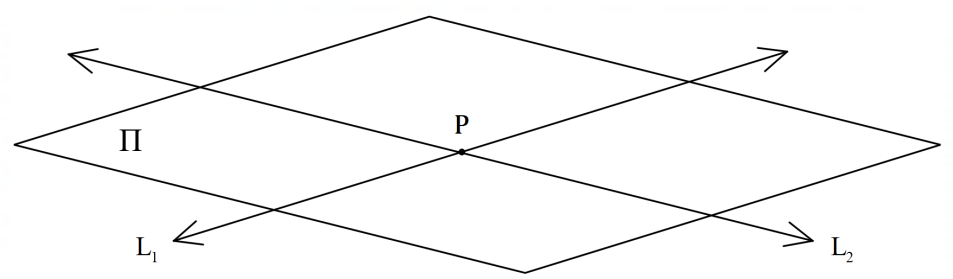
(b) Determine the domain and range for $gf(x)$. (4 marks)

(c) Given that $f^{-1}(x) = 4 - x^2$, explain if it is true that $f^{-1}(-1) = 3$. (2 marks)

See next page

Question 2 (9 marks)

The lines L_1 and L_2 have equations $\boldsymbol{r} = (3 + \lambda)\boldsymbol{i} + (1 + \lambda)\boldsymbol{j} - 2\lambda\boldsymbol{k}$ and $\boldsymbol{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$ respectively, with $a, b, c \in \mathbb{R}$, and they lie on the same plane Π as shown.

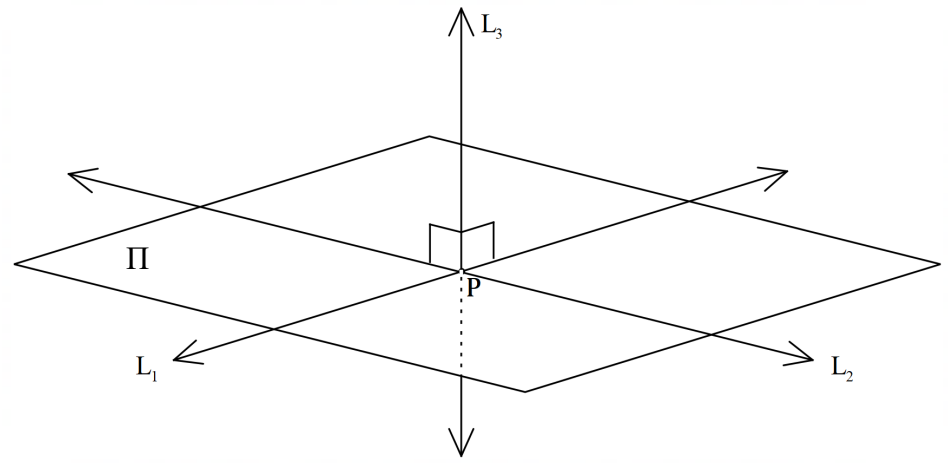


(a) Given that the lines intersect at the point P when $\lambda = 3 = \mu + 2$, determine the value of the constants a, b and c, and the exact distance of point P from the origin.

(4 marks)

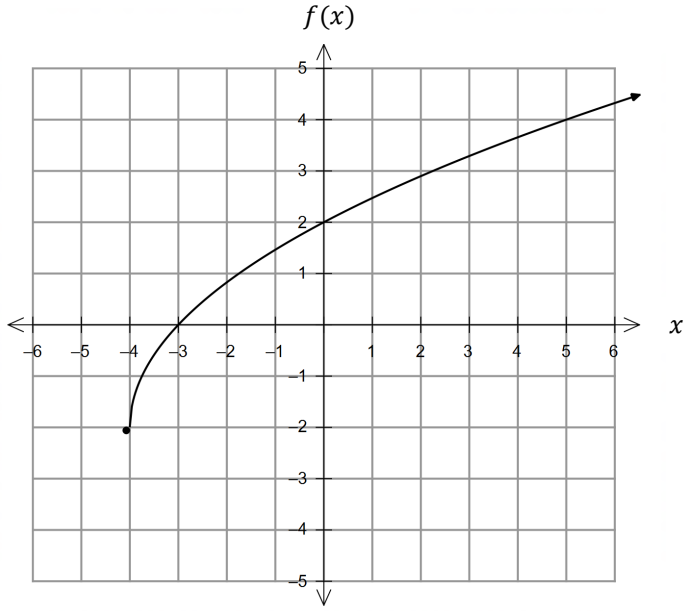
Question 2 continued

(b) A third line L_3 is perpendicular to the plane formed by L_1 and L_2, and passes through P. Determine the vector equation of L_3 and the Cartesian equation of the plane Π. (5 marks)

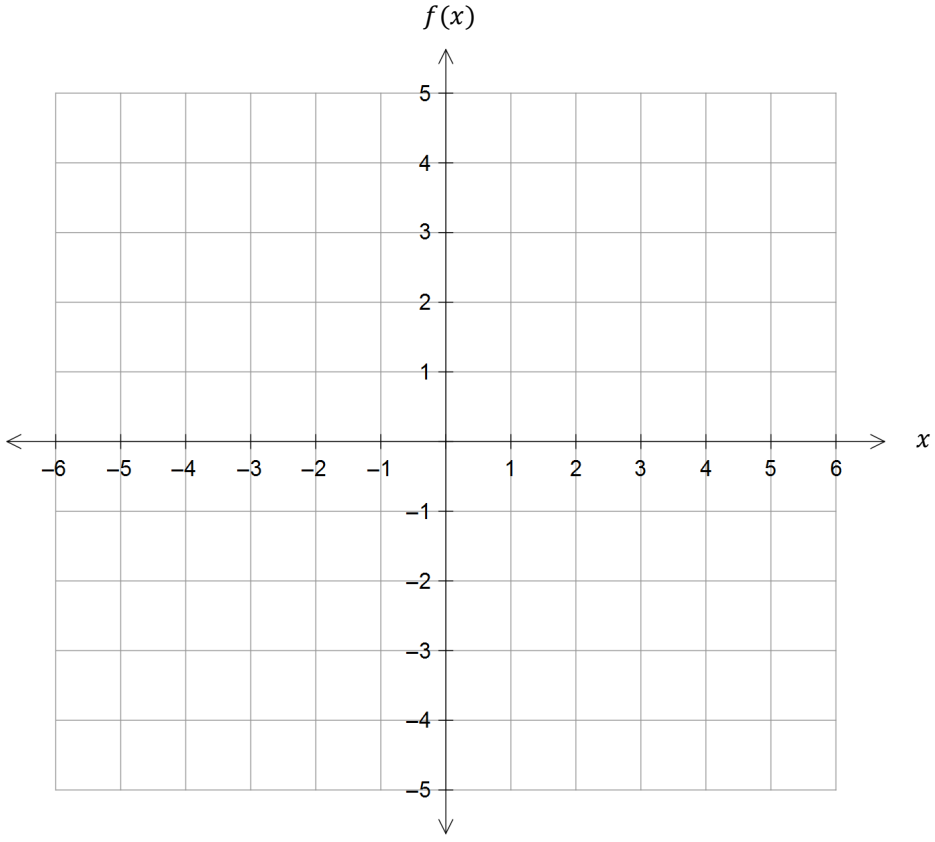


Question 3 **(9 marks)**

The graph of $f(x) = 2\sqrt{x+4} - 2$ is shown below.

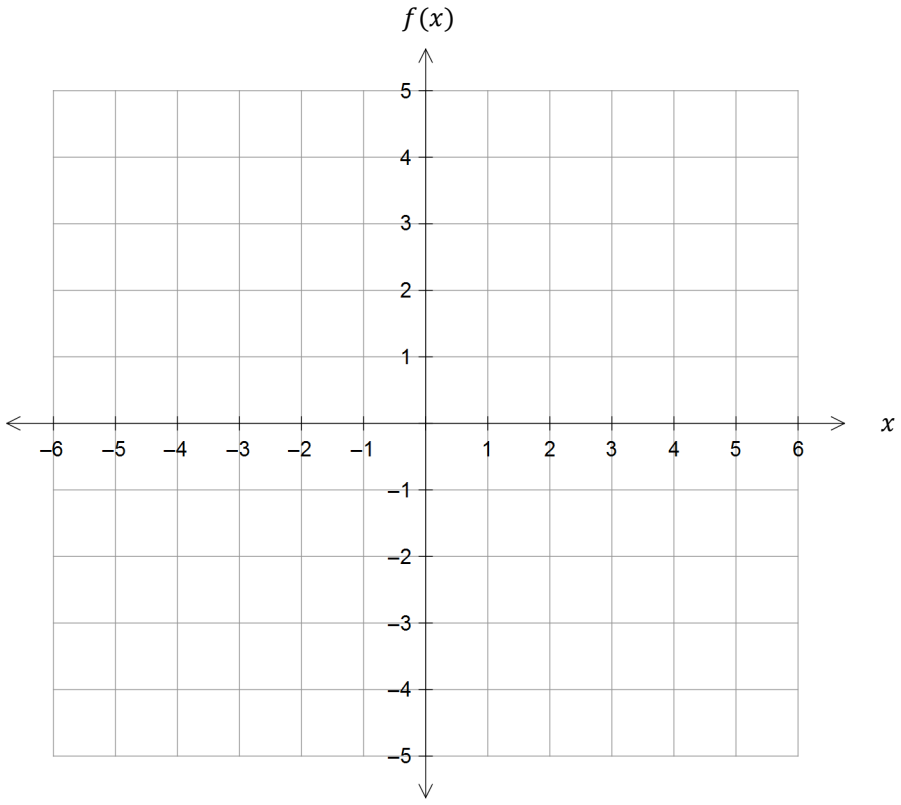


(a) Sketch the graph of $\frac{1}{f(x)}$ on the grid below. (3 marks)



Question 3 continued

(b) Sketch the graph of $|f(-|x|)|$ on the grid below. (3 marks)



(c) The domain of $f(x)$ is restricted to $x \geq k$ so that the inverse of $g(x) = |f(x)|$ exists. Determine the value of k and state the domain and range of $g^{-1}(x)$. (3 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Christ Church Grammar School

2021
TEST 2

MATHEMATICS SPECIALIST Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time for this section:	2 minutes
Working time for this section:	20 minutes
Marks available:	21 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in this assessment

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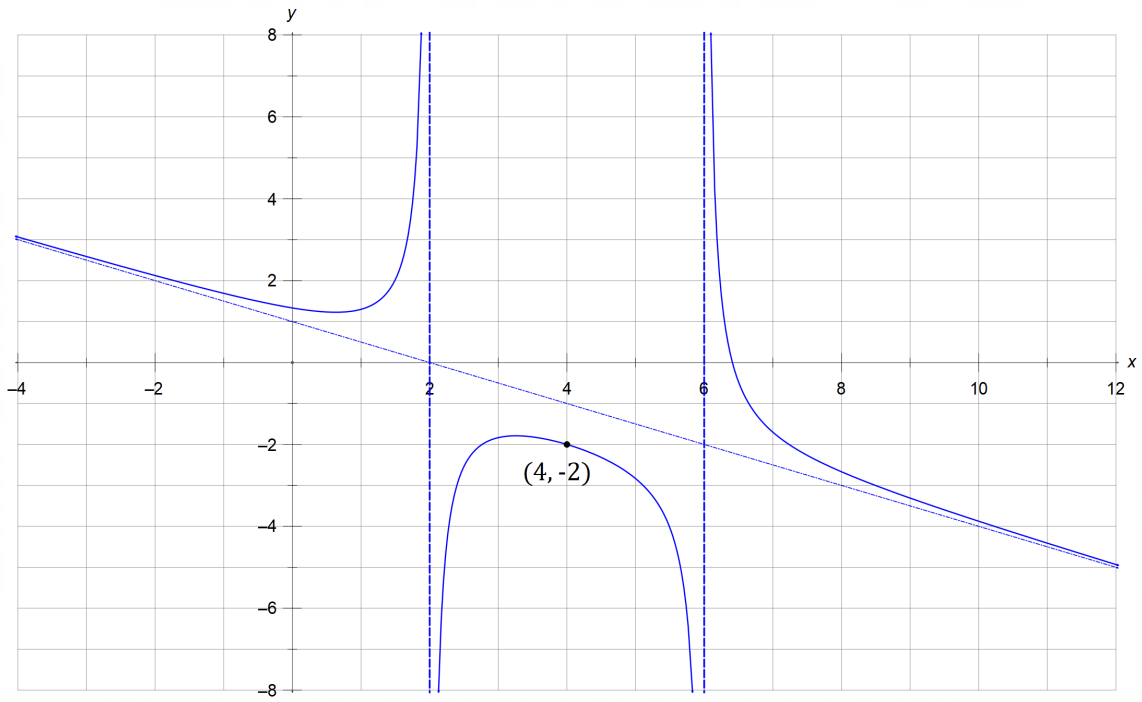
See next page

Question 4 **(5 marks)**

The function $f(x) = ax + b + \frac{k}{(x+c)(x+d)}$ is shown below, where $a, b, c, d, k \in \mathbb{R}$.

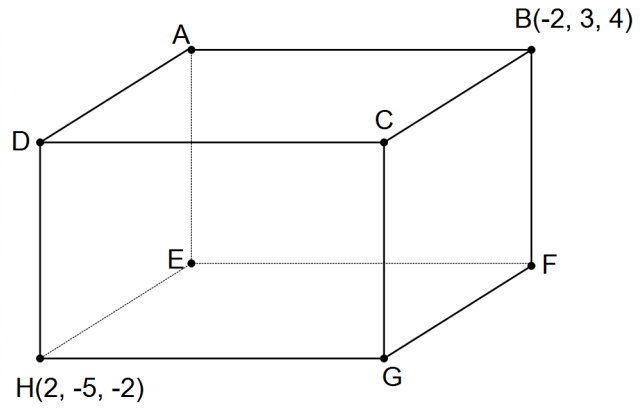


State the value of the constants a, b, c, d and k.

Question 5 **(6 marks)**

The right rectangular prism ABCDEFGH shown is positioned in the Cartesian coordinate system such that AD is parallel to the x-axis, AB is parallel to the y-axis, and EA is parallel to the z-axis.

The vertices B and H have coordinates $(-2, 3, 4)$ and $(2, -5, -2)$ respectively.



(a) State the coordinates of vertex E. (1 mark)

(b) Determine the percentage of the prism that lies in the first octant. (2 marks)

See next page

Question 5 continued

(c) Determine the vector equation of the sphere that has HB as its diameter.

(3 marks)

See next page

Question 6 **(10 marks)**

Triangle *ABC* in space has vertices with position vectors $\overrightarrow{OA} = \boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}$, $\overrightarrow{OB} = -2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\overrightarrow{OC} = 2\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$.

(a) Determine the size of $\angle BAC$ correct to the nearest degree. (3 marks)

(b) Find the vector equation $\boldsymbol{r} = \boldsymbol{p} + \lambda\boldsymbol{d_1} + \mu\boldsymbol{d_2}$ of the plane Π that contains triangle *ABC*. (1 mark)

(c) The line L has equation $\boldsymbol{r} = \alpha\begin{pmatrix} m \\ 7 \\ n \end{pmatrix}$ and it is perpendicular to the plane Π found in part (b).

(i) Show that $m = 1$ and $n = -6$. (2 marks)

See next page

Question 6 continued

(ii) Find the point of intersection between the line L and the plane Π.

(3 marks)

(d) Determine the shortest distance of the plane Π from the origin. (1 mark)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____